Name: ________________________

Math Adventures
Week 5: Number Systems and Parity Bits

A **number system** is a system for expressing numbers.

- Each number system uses a certain **base**, which is the number of different digits available for expressing numbers using that number system.
- We name a number system after the base that it uses.

1. Write out what each number means. For example,
   14,602 = (1 × 10,000) + (4 × 1,000) + (6 × 100) + (0 × 10) + (2 × 1).

   a. 40 =

   b. 128 =

   c. 3,729 =

When expressing a number using a number system other than base 10, we write the base as a subscript on the right side of the number. (If a number does not have a subscript on the right side of it, we assume it is written in base 10.)

2. How do we write 111_2 in base 10?

3. How do we write 77_{10} in binary?

4. Fill in the missing numbers in the following sequence in base 2.

   0_2, 1_2, $\_\_\_\_\_\_\_\__2$, 11_2, $\_\_\_\_\_\_\_\__2$, $\_\_\_\_\_\_\_\__2$, 110_2, 111_2, $\_\_\_\_\_\_\_\__2$, $\_\_\_\_\_\_\_\__2$, 1010_2,

   $\_\_\_\_\_\_\_\__2$, $\_\_\_\_\_\_\_\__2$, 1101_2, $\_\_\_\_\_\_\_\__2$, 1111_2, $\_\_\_\_\_\_\_\__2$, 10001_2, $\_\_\_\_\_\_\_\__2$

5. How do we write 300_8 in base 10?

6. How do we write 26_{10} in base 8?

7. Fill in the missing numbers in the following sequence in base 8.

0_8, 1_8, 2_8, 3_8, $\_\_\_\_\_\_\_\__8$, 5_8, 6_8, $\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$, 11_8, $\_\_\_\_\_\_\_\__8$, 13_8, 14_8, 15_8,

$\_\_\_\_\_\_\_\__8$, 17_8, $\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$, 23_8, $\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$,

$\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$, $\_\_\_\_\_\_\_\__8$

8. How do we write 1100011_2 in base 8?

9. How do we write 765_8 in base 2?

10. How do we write 1001101_2 in base 16?

11. How do we write FF_{16} in base 2?

Sometimes, when computers send data to each other, the data can be corrupted by an interference on the line. Computers use **error-detecting codes** to check whether or not the data has been changed.

- **Parity bits** are one example of error-detecting codes.
- Instead of using all 8 bits of a byte to send a message, a computer can use 7 bits to store the message and 1 bit as a parity bit.

If computers agree to transfer data using an **even parity protocol**, each byte being transferred must contain an **even number of 0s** and an **even number of 1s**.

12. Using an **even parity protocol**, write the parity bit for each byte of code.

    a. 1000101________

    b. 1110010________

    c. 1000111________

13. Using an **even parity protocol**, circle the bytes that have been corrupted.

    10000001 10101010 11110001 01101110

If computers agree to transfer data using an **odd parity protocol**, each byte being transferred must contain an **odd number of 0s** and an **odd number of 1s**

14. Using an **odd parity protocol**, write the parity bit for each byte of code.

    a. 1100011________

    b. 1010111________

    c. 1011011________

15. Using an **odd parity protocol**, circle the bytes that have been corrupted.

    10010001 11101011 11110001 01001110

A computer can send data in a grid format, or in a **parity block**, and add parity bits both horizontally and vertically. This way, if a bit flips during transmission, the receiving computer can tell exactly which one it is and correct it.

16. Using an **even parity protocol**, circle the bit that has flipped.

1	0	0	0	1	0	0	0
0	1	0	1	0	0	1	1
1	1	1	0	1	0	0	0
1	0	0	0	0	0	1	0
0	0	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	1	1	0	0	0	1
0	1	1	1	1	0	1	0

17. Using an **odd parity protocol**, circle the bit that has flipped.

0	1	0	1	1	0	1	1
0	1	1	1	0	1	1	0
0	1	0	0	1	0	0	1
1	1	0	0	0	0	1	0
1	0	1	1	1	1	0	0
0	1	1	1	0	0	1	0
0	0	1	0	0	1	1	0
1	0	1	0	0	0	0	1

Unfortunately, there are some flaws in using parity bits as error detection; for example, if two bits in a byte were to flip during transmission, the receiving computer would not detect a change.

Lesson Summary

A **number system** is a system for expressing numbers.

- Each number system uses a certain **base**, which is the number of different digits available for expressing numbers using that number system.

Base 10:

- There are 10 digits available for expressing numbers: 0-9.
- Each place is 10 times the place to the right of it.

Base 2 (Binary):

- There are 2 digits available for expressing numbers: 0 and 1.
- Each place is 2 times the place to the right of it.
- Binary is the number system used in computers!

Base 8:

- There are 8 digits available for expressing numbers: 0-7.
- Each place is 8 times the place to the right of it.
- Since $2^3 = 8$, base 8 can be used as a condensed representation of binary. One digit in base 8 conveys the same information as three digits in binary!

Base 16 (Hexadecimal):

- There are 16 digits available for expressing numbers: 0-9, A-F.
- Each place is 16 times the place to the right of it.
- Since $2^4 = 16$, base 16 can be used as a condensed representation of binary. One digit in base 16 conveys the same information as four digits in binary!

Computers use **error-detecting codes** to check whether or not the data has been changed by an interference on the line.

- **Parity bits** are one example of error-detecting codes.
- Instead of using all 8 bits of a byte to send a message, a computer can use 7 bits to store the message and 1 bit as a parity bit.
- Before transferring data, computers agree on a **parity protocol** that is either **odd** or **even**.
- A computer counts the number of 0s and the number of 1s in the message it wants to send and decides to store either 0 or 1 in the parity bit to match the parity protocol.
- If a bit is flipped during transmission, the receiving computer will detect that the byte does not match the parity protocol and ask for the message to be sent again.

References: Khan Academy, *Hard Math for Elementary School* by Glenn Ellison, Computer Science GCSE GURU