

Name: \_\_\_\_\_

**Math Adventures**  
**Week 8: Flexagons**

**Topology** is a branch of math that studies the properties of an object that stay the same after the object is deformed.

A **flexagon** is a folded figure that can be “flexed” along its folds to reveal different faces.

- A **tetraflexagon** is a flexagon in the shape of a rectangle.
- A **hexaflexagon** is a flexagon in the shape of a hexagon.

**Tri-Tetraflexagon:**

1. Cut out the tri-tetraflexagon template. How many small square panels does the template have? (Do not count the two panels that are marked “GLUE.” We will glue these two panels together later.)

Make a tri-tetraflexagon.

2. How many large square faces (each with four panels) do you see on the tri-tetraflexagon?
3. How many small square panels do you see on both sides of the tri-tetraflexagon?

Flex the tri-tetraflexagon! Hold the flexagon so that face 1 is facing you. Fold the flexagon in half vertically so that face 1 is on the outside of the fold. Pull the flexagon open from the fold line. In order to flex again, make sure that face 1 is facing you.

4. What happens when you flex the tri-tetraflexagon?
5. Why do you think this happens? Where were the other small square panels?

6. How many large square faces does the tri-tetraflexagon have?
7. Where do you think the tri- prefix in “tri-tetraflexagon” comes from?

With a pencil, draw a circle at the center of both visible large square faces so that part of the circle is on each small square panel.

8. After flexing the tri-tetraflexagon a few times, what happens to the circles you drew?
9. How many times do you have to flex the tri-tetraflexagon to return to the original orientation of a face?
10. How many combinations of faces (on both sides) does the tri-tetraflexagon have?

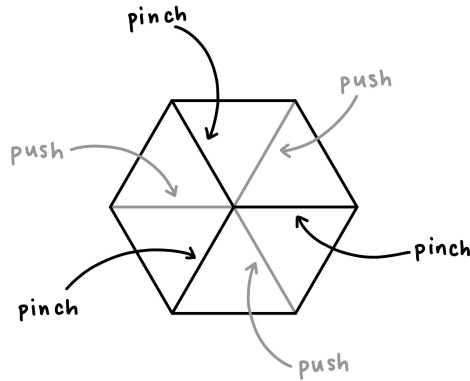
### **Tri-Hexaflexagon:**

11. Cut out the tri-hexaflexagon template. How many small triangular panels does the template have? (Do not count the two panels that are marked “GLUE.” We will glue these two panels together later.)

Make a tri-hexaflexagon.

12. How many large hexagonal faces (each with six panels) do you see on the tri-hexaflexagon?
13. How many small triangular panels do you see on both sides of the tri-hexaflexagon?

Flex the tri-hexaflexagon! Push in on the three symmetric gray lines (creases in the paper) while pinching the three other black lines (borders of the paper) at the same time. Pull the trihexaflexagon open from the top.



14. What happens when you flex the tri-hexaflexagon?

15. How many large hexagonal faces does the tri-hexaflexagon have?

With a pencil, draw a circle at the center of both visible large hexagonal faces so that part of the circle is on each small triangular panel.

16. After flexing the tri-hexaflexagon a few times, what happens to the circles you drew?

17. How many times do you have to flex the tri-hexaflexagon to return to the original orientation of a hexagonal face?

18. How many combinations of faces (on both sides) does the tri-hexaflexagon have?

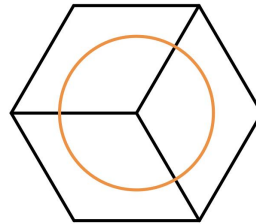
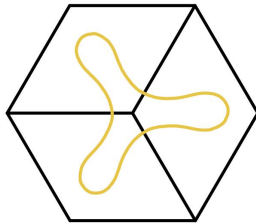
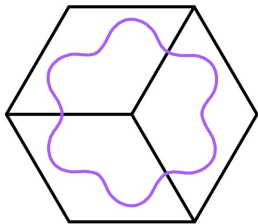
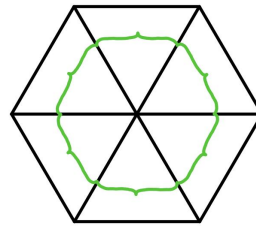
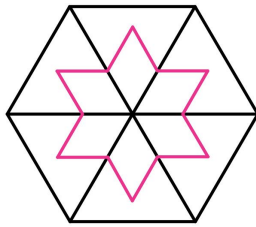
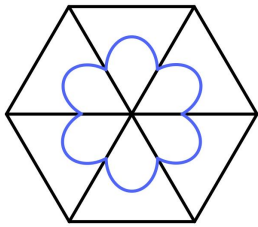
Some flexagons are topologically equivalent to Möbius strips! For example, a tri-hexaflexagon is topologically equivalent to a Möbius strip with three  $180^\circ$  twists.

**Hexa-Hexaflexagon:**

19. How many large hexagonal faces do you think a hexa-hexaflexagon has?

Make a hexa-hexaflexagon! Feel free to further decorate the faces if you'd like.

20. Try to flex the hexa-hexaflexagon to find all six of the faces below! Circle the ones that you find.



References: Olga Radko Endowed Math Circle Archive, Hexaflexagon Toolkit, Aunt Annie's Crafts